

Exercise 12

Task 1

Let $\mathcal{G} = (V, E)$ be a graph, where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. We consider \mathcal{G} as a structure with universe V and binary relation E . Formulate the following statements as MSO-formulas:

- (a) The graph is strongly connected.
- (b) The graph is bipartite (= the underlying undirected graph is bipartite).
- (c) The graph is a tree with a root.

Task 2

Find MSO-formulas for the following regular languages:

- (a) $L_1 = L((a|b)^*a)$
- (b) $L_2 = \{w \in \Sigma^+ \mid w \text{ begins and ends with } b\}$
- (c) $L_3 = L(b(a|b)^*b)$