

Exercise 13

Task 1

Find an MSO-formula corresponding to the regular language

$$L = \{w \in \Sigma^+ \mid \text{The number of } a\text{'s in } w \text{ is odd}\}.$$

Hint: Consider the technique from the proof of Büchi's Theorem.

Task 2

Which regular languages over $\Sigma = \{a, b, c\}$ correspond to the following MSO formulas?

(a) $\forall x \forall y (P_a(x) \wedge P_b(y) \wedge (x < y) \wedge (\forall z (x < z < y) \rightarrow \neg P_b(z)))$
 $\rightarrow (\exists x_1 \exists x_2 (x < x_1 < x_2 < y) \wedge P_c(x_1) \wedge P_c(x_2))$

(b) $\exists X (\exists x \exists y (\forall u (x \leq u \leq y) \wedge x \in X \wedge y \in X) \wedge$
 $\forall x \forall y (y = x + 1 \rightarrow (x \in X \leftrightarrow \neg(y \in X))))$