

Exercise 5

Task 1

Let $\Sigma = \{a, b\}$. We consider the following linear orders on Σ^* : The *lexicographic order* $\leq_{\text{lex}}$ is defined by

$$u \leq_{\text{lex}} v \iff u \text{ is a prefix of } v \text{ or} \\ \text{there are } x, y, z \in \Sigma^* \text{ such that } u = xay \text{ and } v = xbz$$

and the *length-lexicographic order* $\leq_{\text{llex}}$ is defined by

$$u \leq_{\text{llex}} v \iff |u| < |v| \text{ or } (|u| = |v| \text{ and } u \leq_{\text{lex}} v).$$

Show that the relations $\leq_{\text{lex}}$ and $\leq_{\text{llex}}$ are synchronously rational.

Task 2

Let $\Sigma = \{a, b\}$ and let $n \geq 1$. Show that the language

$$\{w_1 \otimes \cdots \otimes w_n \mid w_1, \dots, w_n \in \Sigma^*\} \subseteq (\Sigma_{\#}^n)^*$$

is regular by constructing a finite automaton for this language.