a)

$$\begin{array}{r} \left(-3x^3-13x^2+15x\ +25\right) \\ \underline{\ 3x^3\ -7x^2\ -\frac{5}{3}x\ +\frac{25}{3}\ } \\ -20x^2+\frac{40}{3}x+\frac{100}{3} \end{array} \Big/ \left(9x^3-21x^2-5x+25\right) = -\frac{1}{3}+\frac{-20x^2+\frac{40}{3}x+\frac{100}{3}}{9x^3-21x^2-5x+25}$$

div : $-\frac{1}{3}$

mod: $-20x^2+\frac{40}{3}x+\frac{100}{3}$

$$\begin{array}{r} \left(\ \ 9x^3-21x^2\ \ -5x+25\right) \\ \underline{-9x^3\ \ +6x^2+15x\ \ \ \ \ } \\ -15x^2+10x+25 \\ \underline{\ \ 15x^2-10x-25} \\ 0 \end{array} \Big/ \left(-20x^2+\frac{40}{3}x+\frac{100}{3}\right) = -\frac{9}{20}x+\frac{3}{4}$$

ggT: $(-20x^2+\frac{40}{3}x+\frac{100}{3})/(-20) = x^2-\frac{2}{3}x-\frac{5}{3}$

b)

$$\begin{array}{r}
\big(-20x^6 + x^5 - 42x^4 + 10x^3 - 49x^2 - 6x - 24\big) / \big(-20x^3 - 15x^2 - 30x\big) = x^3 - \tfrac{4}{5}x^2 + \tfrac{6}{5}x - \tfrac{1}{5} + \dfrac{-16x^2 - 12x - 24}{-20x^3 - 15x^2 - 30x} \\
\underline{20x^6 + 15x^5 + 30x^4} \\
16x^5 - 12x^4 + 10x^3 \\
\underline{-16x^5 - 12x^4 - 24x^3} \\
-24x^4 - 14x^3 - 49x^2 \\
\underline{24x^4 + 18x^3 + 36x^2} \\
4x^3 - 13x^2 - 6x \\
\underline{-4x^3 - 3x^2 - 6x} \\
-16x^2 - 12x
\end{array}$$

div : $x^3 - \frac{4}{5}x^2 + \frac{6}{5}x - \frac{1}{5}$

mod: $-16x^2 - 12x - 24$

$$\begin{array}{r}
\big(-20x^3 - 15x^2 - 30x\big) / \big(-16x^2 - 12x - 24\big) = \tfrac{5}{4}x \\
\underline{20x^3 + 15x^2 + 30x} \\
0
\end{array}$$

ggT: $(-16x^2 - 12x - 24)/(-16) = x^2 + \frac{3}{4}x + \frac{3}{2}$

c)

$$\begin{array}{r} \big(\quad x^3 - 3x^2 + 5x - 3\big) \,/\, \big(x^3 - 1\big) = 1 + \dfrac{-3x^2 + 5x - 2}{x^3 - 1} \\ \underline{-\,x^3 \qquad\qquad + 1} \\ -\,3x^2 + 5x - 2 \end{array}$$

div : 1

mod: $-3x^2 + 5x - 2$

$$\begin{array}{r} \big(\quad x^3 \qquad\qquad\quad - 1\big) \,/\, \big(-\,3x^2 + 5x - 2\big) = -\,\tfrac{1}{3}x - \tfrac{5}{9} + \dfrac{\frac{19}{9}x - \frac{19}{9}}{-3x^2 + 5x - 2} \\ \underline{-\,x^3 + \tfrac{5}{3}x^2 \;\; - \tfrac{2}{3}x} \\ \tfrac{5}{3}x^2 \;\; - \tfrac{2}{3}x \;\; - 1 \\ \underline{-\,\tfrac{5}{3}x^2 + \tfrac{25}{9}x - \tfrac{10}{9}} \\ \tfrac{19}{9}x - \tfrac{19}{9} \end{array}$$

$$\begin{array}{r} \big(-\,3x^2 + 5x - 2\big) \,/\, \big(\tfrac{19}{9}x - \tfrac{19}{9}\big) = -\,\tfrac{27}{19}x + \tfrac{18}{19} \\ \underline{\quad 3x^2 - 3x} \\ 2x - 2 \\ \underline{-\,2x + 2} \\ 0 \end{array}$$

ggT: $(\frac{19}{9}x - \frac{19}{9})/(\frac{19}{9}) = x - 1$