

Exercise 1

Task 1 (Landau symbols)

True or false? Give a reason for your answer.

- (a) $n \log n \in \mathcal{O}(n^2)$
- (b) $n^n \in \mathcal{O}(2^{n^2})$
- (c) $n - \log n \in o(n)$
- (d) $2 + (-1)^n \in \Theta(1)$
- (e) $n! \in \omega(2^n)$
- (f) $\exists p > 0 : 2^{(\log n)^2} \in \mathcal{O}(n^p)$

Task 2 (Mastertheorem)

What is the asymptotic growth of the following recursive functions?

- (a) $T_1(n) = 7 \cdot T_1\left(\frac{n}{2}\right) + 4n$
- (b) $T_2(n) = 7 \cdot T_2\left(\frac{n}{2}\right) + 1000n^2$
- (c) $T_3(n) = 8 \cdot T_3\left(\frac{n}{2}\right) + n^2$
- (d) $T_4(n) = 8 \cdot T_4\left(\frac{n}{2}\right) + n^3$
- (e) $T_5(n) = 6 \cdot T_5\left(\frac{n}{3}\right) + n^3$
- (f) $T_6(n) = 64 \cdot T_6\left(\frac{n}{8}\right) + n^2$

Task 3 (Mergesort)

- (a) Sort the array $[2, 8, 13, 4, 7, 16, 3, 12]$ with Mergesort.
- (b) Assume Mergesort divides the current array into k subarrays of roughly same size and recursively sort the k arrays. What happens with the asymptotic runtime of Mergesort considering this modification?

Task 4 (Divide & Conquer-Multiplikation)

Calculate $2063 \cdot 3201$ with the algorithm of Karatsuba

Hint: You do not have to use base 2.