Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey

Excercise 4

Task 1
Sort the array $[3, 5, 2, 1, 4]$ with Bottom-up Heapsort. Encode the sink paths of the algorithm as explained in the lecture (slides 64, 65):
For a bitstring $a_1 \dots a_t \in \{0,1\}^*$, we have

$$c_1(a_1 \dots a_t) = a_1 0 a_2 0 \dots a_{t-1} 0 a_t 1$$

and

$$c_2'(a_1 \dots a_t) = c_1(\text{bin}(\lceil \log_2(n) \rceil - t)) a_1 \dots a_t.$$

Task 2

(a) Show that the leaves of a heap of size n are at positions

$$\lfloor n/2 \rfloor + 1, \lfloor n/2 \rfloor + 2, \dots, n$$

of the array representation.

(b) How many comparisons do Heapsort need on a sorted list?

(c) How many comparisons do Heapsort need on a reversed sorted list?

Task 3
Show that a heap of size n has at most $\lceil n/2^{h+1} \rceil$ nodes of height h (the height of a node is the height of the subtree rooted at this node, i.e., the maximal length of a path from this node to a leaf).

Task 4
For an array of size $n = 6$, Heapsort gives the following encodings (using c_2') of the sink paths:

$$10010100111110100111001010011100 10.$$

What was the input array?