Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey

Exercise 7

Task 1 (Slide 158)
Let A be the adjacency matrix of a directed graph with nodes $V = \{1, \dots, n\}$. Show for every $k \geq 1$ and every $i, j \in V$ that $A^k(i, j) = 1$ if and only if there is a path of length k from i to j. Addition is done using $\vee$ and multiplication is done using $\wedge$.

Task 2
A literal is a variable A or its negation $\neg A$. The *negation* of a literal $\overline{L}$ is defined as: If $L = A$ then $\overline{L} = \neg A$, and if $L = \neg A$ then $\overline{L} = A$. Let F be a propositional formula in *negation normal form* (meaning that $\neg$ may only be directly applied to variables). A literal L is *pure* in F if F does not contain $\overline{L}$. Show that if F is satisfiable then there is also a variable assignment that makes L true and still satisfies F.

Task 3
Let 1 resp. 0 be propositional formulas that are always true resp. false. For a variable A and a propositional formula F we define $F|_A$ by induction as follows:

- $A|_A = 1$ and $B|_A = B$ for all $B \neq A$.
- $\neg F|_A = \begin{cases} \neg(F|_A) & \text{if } F \neq A, \\ 0 & \text{otherwise.} \end{cases}$
- $(F_1 \wedge F_2)|_A = (F_1|_A) \wedge (F_2|_A)$
- $(F_1 \vee F_2)|_A = (F_1|_A) \vee (F_2|_A)$

Show for every propositional formula F and every variable A that $A \wedge F \equiv A \wedge F|_A$.

Task 4
Show that SAT $\leq_p$ 3$-$SAT.

Task 5
Show that every undirected graph in which every node has degree 2 is the union of disjoint cycles.