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Exercise 2

Task 1
Calculate $2063 \cdot 3201$ using the algorithm of Karatsuba. You do not have to use base 2.

Task 2
Use the algorithm of Strassen to calculate the following matrix product:

$$\begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

Task 3
Professor Caesar wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's algorithm. His algorithm will use the divide-and-conquer method, dividing each matrix into pieces of size $n/4 \times n/4$, and the divide and combine steps together will take $\Theta(n^2)$ time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen's algorithm. If his algorithm creates a subproblems then the recurrence for the running time $T(n)$ becomes $aT(n/4) + \Theta(n^2)$. What is the largest integer value for a for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm?

Task 4
Show that a binary tree with N leaves has at least height $\log_2(N)$.

Task 5
Sort the array $[2, 8, 13, 5, 7, 16, 3, 12]$ using Quicksort.