

## Exercise 3

### Task 1

Sort the array

$$[7, 3, 8, 1, 5, 2, 4, 6]$$

using Standard Heapsort and then sort it using Bottom-up Heapsort. How many comparisons do you need in each case?

### Task 2

Show Jensen's inequality (slide 8).

### Task 3 (Slides 53 and 58)

Show that for the  $n$ -th harmonic number  $H_n$  the following inequalities hold:

$$\ln(n+1) \leq H_n \leq \ln(n) + 1.$$

*Hint:*  $\ln(n) = \int_1^n \frac{1}{x} dx$ .

### Task 4 (Slide 77)

Let  $n \in \mathbb{N}$ . Show that the function  $f(x) = \log_2(\log_2(n) - x)$  is concave on  $(-\infty, \log_2(n))$ .