Exercise 4

Task 1

(a) Show that the leaves of a heap of size n are at positions

 $\lfloor n/2 \rfloor + 1, \lfloor n/2 \rfloor + 2, \dots, n$

of the array representation.

- (b) How many comparisons does build-heap need on a sorted list?
- (c) How many comparisons does build-heap need on a reversed sorted list?

Task 2

Sort the following list via Radixsort.

[224, 421, 319, 121, 914, 314]

Task 3

Show that the median of five numbers can be computed using six comparisons.

Task 4

Does the algorithm "Median of the Medians" run in linear time, if one uses blocks of three or blocks of seven?

Task 5

Which of the following pairs is a subset system, respectively matroid?

- (a) $(\{1, 2, 3\}, \{\emptyset, \{1\}, \{3\}, \{1, 2\}\})$
- (b) $(\{1,2,3\}, \{\emptyset, \{1\}, \{2\}, \{3\}, \{2,3\}\})$
- (c) (E, U), where E is a finite set and $U = \{A \subseteq E \mid |A| \le k\}$ for a $k \in \mathbb{N}$.
- (d) (E, U), where E is a finite set, $\{E_i \mid 1 \le i \le k\}$ is a partition of E and

$$U = \{ A \subseteq E \mid |A \cap E_i| \le 1 \text{ for all } 1 \le i \le k \}.$$