## Exercise 5

## Task 1

Compute a spanning subtree of maximal weight using Kruskal's algorithm for the following graph:


## Task 2

(a) Show that for each tree $T=(V, E)$ with $|V|>0$ we have $|E|=|V|-1$.
(b) Show that every connected graph has a spanning subtree.

## Task 3

Use Dijkstra's algorithm to compute all shortest paths starting at node $s$.


## Task 4

Let $F_{n}$ be the $n$-th Fibonacci number $\left(F_{1}=F_{2}=1\right.$ and $\left.F_{n+1}=F_{n}+F_{n-1}\right)$. Show that

$$
\sum_{i=1}^{n} F_{i}^{2}=F_{n} \cdot F_{n+1}
$$

and

$$
\sum_{i=1}^{2 n+1}(-1)^{i-1} F_{i}=F_{2 n}+1
$$

