Exercise 5

Task 1

Compute a spanning subtree of maximal weight using Kruskal's algorithm for the following graph:



Task 2

(a) Show that for each tree T = (V, E) with |V| > 0 we have |E| = |V| - 1.

(b) Show that every connected graph has a spanning subtree.

Task 3

Use Dijkstra's algorithm to compute all shortest paths starting at node s.



Task 4

Let F_n be the *n*-th Fibonacci number $(F_1 = F_2 = 1 \text{ and } F_{n+1} = F_n + F_{n-1})$. Show that

$$\sum_{i=1}^{n} F_i^2 = F_n \cdot F_{n+1}$$

and

$$\sum_{i=1}^{2n+1} (-1)^{i-1} F_i = F_{2n} + 1.$$