Exercise 1

Task 1
Prove that the *Vandermonde*-Matrix

$$V(a_0,\dots,a_{n-1}) = \begin{pmatrix} 1 & a_0 & a_0^2 & \dots & a_0^{n-1} \\ 1 & a_1 & a_1^2 & \dots & a_1^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & a_{n-1} & a_{n-1}^2 & \dots & a_{n-1}^{n-1} \end{pmatrix}$$

is ivertible if and only if the numbers $a_0,\dots,a_{n-1}$ are pairwise different.
Hint: Show first that the following equation holds:

$$\det V(a_0,\dots,a_{n-1}) = \prod_{0\le i<j<n} (a_j - a_i)$$

Task 2 (Fast Fourier Transform)

(a) Use the FFT to compute the discrete Fourier transform of the polynomial $f(x) = x + 2x^2 + 3x^3$ over $\mathbb{C}$.

(b) Compute $(x+2)\cdot(2x-1)$ with the FFT.

Task 3
Let $A, B \subseteq \{1,\dots,10n\}$ be sets with $|A| = |B| = n$. We want to compute

$$C := \{a + b : a \in A, b \in B\}$$

and the number of possibilities to write $c \in C$ as a sum of elements in A and B. Specify an algorithm that solves the problem in time $\mathcal{O}(n \log n)$.