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Exercise 4

Task 1
Consider the following algorithm, which tests probabilistically if $AB = C$ for given matrices $A, B, C \in \mathbb{Z}^{n \times n}$:

1. Choose a vector $v \in \{0,1\}^{n \times 1}$ randomly and uniformly distributed.
2. Compute $w = A(Bv) - Cv$.
3. If $w = 0$ return "yes", otherwise "no".

Prove that in the case $AB \neq C$ the algorithm returns "yes" with a probability of at most $\frac{1}{2}$.

Task 2
Let $G = (V, E)$ be an undirected graph with

$$V = \{1,2,3,4,5,6\}, \quad E = \{\{1,3\},\{1,6\},\{2,3\},\{2,5\},\{3,5\},\{4,6\},\{5,6\}\}.$$

(a) Compute the Tutte matrix T_G of G.

(b) Compute the polynomial $\det(T_G)$.

(c) Does G have a perfect matching? If yes, name all perfect matchings of G. If no, justify your answer.

Task 3
Let $G = (V, E)$ be an undirected graph with $V = \{1, \ldots, n\}$. Let $T^G = (T_{u,v})_{1 \le u,v \le n}$ be the matrix defined by

$$T_{u,v} = \begin{cases} x_{u,v} & \text{if } \{u,v\} \in E, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Let G be a bipartite graph. This means, there are disjoint subsets $U, W \subset V$ such that $V = U \cup W$ and $\{u, w\} \in E$ only if $u \in U$ and $w \in W$ (or $w \in U$ and $u \in W$). Show that G has a perfect matching if and only if $\det(T^G) \neq 0$.

(b) Does (a) also hold, if G is not bipartite?