

Exercise 5

Task 1

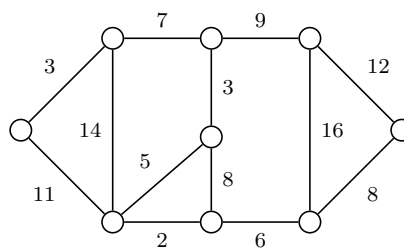
Which of the following pairs is a subset system, respectively matroid?

- (a) $(\{1, 2, 3\}, \{\emptyset, \{1\}, \{3\}, \{1, 2\}\})$
- (b) $(\{1, 2, 3\}, \{\emptyset, \{1\}, \{2\}, \{3\}, \{2, 3\}\})$
- (c) (E, U) , where E is a finite set and $U = \{A \subseteq E \mid |A| \leq k\}$ for a $k \in \mathbb{N}$.
- (d) (E, U) , where E is a finite set, $\{E_i \mid 1 \leq i \leq k\}$ is a partition of E and

$$U = \{A \subseteq E \mid |A \cap E_i| \leq 1 \text{ for all } 1 \leq i \leq k\}.$$

Task 2

Compute a spanning subtree of maximal weight using Kruskal's algorithm for the following graph:



Task 3

- (a) Show that for each tree $T = (V, E)$ with $|V| > 0$ we have $|E| = |V| - 1$.
- (b) Show that every connected graph has a spanning subtree.

Task 4

Use Dijkstra's algorithm to compute all shortest paths starting at node s .

