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Exercise 6

Task 1

Let F_n be the n-th Fibonacci number ($F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$). Show that

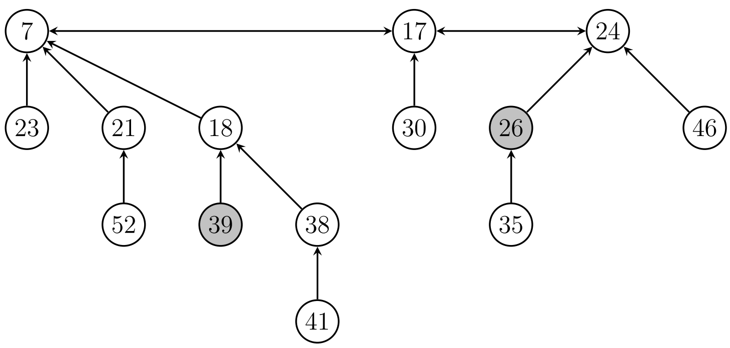
$$\sum_{i=1}^{n} {F_i}^2 = F_n \cdot F_{n+1}$$

and

$$\sum_{i=1}^{2n+1} (-1)^{i-1} F_i = F_{2n} + 1.$$

Task 2

Given the following Fibonacci heap:



Perform the following operations in that order:
delete-min, **decrease-key**(“52”, 9), **decrease-key**(“46”, 3), **insert**(42), **delete-min**, **decrease-key**(“35”, 7)

Task 3

Show Theorem 17 from the lecture: For all $k \geq 0$ we have

$$F_k = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^{k+1} - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^{k+1}.$$

In Task 1 we used $F_0 = 0$ and $F_1 = 1$, but here we use $F_0 = F_1 = 1$!

Task 4

Prove or disprove: The height of a Fibonacci heap of size n is at most $O(\log n)$.