Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey



# Exercise 4

**Task 1**
Consider the following algorithm, which tests probabilistically if $AB = C$ for given matrices $A, B, C \in \mathbb{Z}^{n\times n}$:

1. Choose a vector $v \in \{0,1\}^{n\times 1}$ randomly and uniformly distributed.
2. Compute $w = A(Bv) - Cv$.
3. If $w = 0$ return "yes", otherwise "no".

Prove that in the case $AB \neq C$ the algorithm returns "yes" with a probability of at most $\frac{1}{2}$.

**Task 2**
Let $G = (V, E)$ be an undirected graph with

$$V = \{1,2,3,4,5,6\}, \quad E = \{\{1,3\},\{1,6\},\{2,3\},\{2,5\},\{3,5\},\{4,6\},\{5,6\}\}.$$

(a) Compute the Tutte matrix $T_G$ of $G$.

(b) Compute the polynomial $\det(T_G)$.

(c) Does $G$ have a perfect matching? If yes, name all perfect matchings of $G$. If no, justify your answer.

**Task 3**
Let $G = (V, E)$ be an undirected graph with $V = \{1, \ldots, n\}$. Let $T^G = (T_{u,v})_{1\le u,v\le n}$ be the matrix defined by

$$T_{u,v} = \begin{cases} x_{u,v} & \text{if } \{u,v\} \in E, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Let $G$ be a bipartite graph. This means, there are disjoint subsets $U, W \subset V$ such that $V = U \cup W$ and $\{u, w\} \in E$ only if $u \in U$ and $w \in W$ (or $w \in U$ and $u \in W$). Show that $G$ has a perfect matching if and only if $\det(T^G) \neq 0$.

(b) Does (a) also hold, if $G$ is not bipartite?