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Lehrstuhl Theoretische Informatik
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Exercise 1

Task 1
Prove or disprove the following statements:

(a) $n \log n \in \mathcal{O}(\sqrt{n} \cdot n)$

(b) $n^{k+1} \in \mathcal{O}(n^k)$, $k \in \mathbb{N}$

(c) $n^n \in \Omega(5^n)$

(d) $n - \log n \in o(n)$

(e) $k + \ell \cdot (-1)^n \in \Theta(1)$, $k, \ell \in \mathbb{N}$, $k > \ell$

(f) $n^3 \in \omega(n^2 \sin(n!))$

Task 2
Let $f \colon \mathbb{N} \to \mathbb{N}$ with $f(n) \in \Theta(n)$. Prove or disprove the following statements:

(a) $f(n)^k \in \Theta(n^k)$ for all $k \in \mathbb{N}$, $k \geq 1$

(b) $3^{f(n)} \in \Theta(3^n)$

Task 3
Use the Master Theorem to determine the asymptotic growth of the following functions:

(a) $T_1(n) = 99 \cdot T_1\left(\frac{n}{100}\right) + 24n$

(b) $T_2(n) = 10 \cdot T_2\left(\frac{n}{3}\right) + 1000n^2$

(c) $T_3(n) = 16 \cdot T_3\left(\frac{n}{2}\right) + n^4$

(d) $T_4(n) = 16 \cdot T_4\left(\frac{n}{2}\right) + n^3$

Task 4

Give a recursive equation for the running time of the algorithm for f. Use the Master Theorem I to compute the running time of the algorithm.

function $f(n : \text{integer})$
  let m be the smallest number of the form 3^k with $3^k \geq n$;
  if $m = 1$ **then print**(goodbye)
  else
    for $i = 1$ to m^2 **do**
      print(hello)
    endfor
    $m := m/3$;
    for $i = 1$ to 8 **do**
      $f(m)$
    endfor
  endif
endfunction