# Exercise 2

**Task 1**
Sort the array $[4, 19, 8, 1, 18, 7, 11, 13]$ using Mergesort.

**Task 2**
Calculate $1295 \cdot 4077$ using the algorithm of Karatsuba. You do not have to use base 2.

**Task 3**
Use the algorithm of Strassen to calculate the following matrix product:

$$\begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 8 \\ -5 & 1 \end{pmatrix}$$

**Task 4**
Professor Caesar wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's algorithm. His algorithm will use the divide-and-conquer method, dividing each matrix into pieces of size $n/8 \times n/8$, and the divide and combine steps together will take $\Theta(n^2)$ time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen's algorithm. If his algorithm creates $a$ subproblems then the recurrence for the running time $T(n)$ becomes $aT(n/8) + \Theta(n^2)$. What is the largest integer value for $a$ for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm?

**Task 5**
Show that a binary tree with $N$ leaves has at least height $\log_2(N)$.