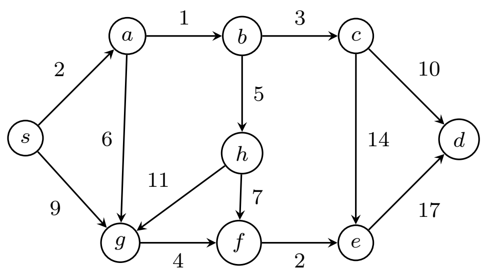
Exercise 6

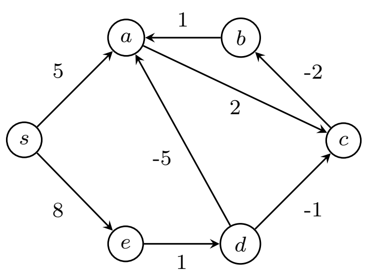
Task 1
Use Dijkstra's algorithm to compute all shortest paths starting at node s. Show the values of the program variables B, R, U, p, D after each iteration of the main **while**-loop of Dijkstra's algorithm.



Task 2
In this task we want to consider directed graphs with possible negative-weighted edges. In many cases Dijkstra's algorithm will not yield a correct result.

(a) Assuming that all nodes are reachable from the source node s, what other necessary condition has to be assumed to guarantee the existence of shortest paths to each node?

(b) Consider the following graph:



Why does Dijkstra's algorithm not work in this case (source node s), even though shortest paths exist to each node of the graph?

(c) Modify Dijkstra's algorithm, such that for every weighted directed graph with source node s one always obtains the shortest path to each node (assuming their existence). What is the running time of your algorithm?

(d) Test your algorithm on the example of part b. The distance variable is sufficient.

Task 3

Show Theorem 17 from the lecture (slide 155) : For all $k \geq 0$ we have

$$F_k = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^{k+1} - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^{k+1}.$$

Here we use $F_0 = F_1 = 1$ (there are other conventions).