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Exercise 4

Task 1 (Nondeterministic Logspace)**.** A *cycle* in a directed graph is a path $v_1, \ldots, v_n$, such that $v_1 = v_n$ and $v_i \neq v_j$ for all other nodes v_i and v_j. A directed graph is called *acyclic*, if it does not contain a cycle. Does the problem ACYCLIC
Input: A directed graph $G = (V, E)$.
Question: Is G acyclic?
belong to the complexity class **NL**?

Task 2. Let

$$\mathbf{EXP} = \bigcup_{k \in \mathbb{N}} \mathbf{DTIME}(2^{n^k}) \text{ and } \mathbf{NEXP} = \bigcup_{k \in \mathbb{N}} \mathbf{NTIME}(2^{n^k}).$$

Prove the following statement by using the padding technique: If $\mathbf{P} = \mathbf{NP}$, then also $\mathbf{EXP} = \mathbf{NEXP}$.

Task 3. Show that $\mathbf{NL} = \mathbf{coNL}$ is equivalent to the theorem of Immerman and Szelepcsényi.

Task 4. Let $f(n) = n^k$. Show for $L \neq \Sigma^*$ and $L \neq \varnothing$:

$$\mathsf{Pad}_f(L) \leq_m^{\log} L.$$