Exercise 6

Task 1.

(a) Let DNF-SAT be the set of satisfiable propositional formulas in disjunctive normal form. Find an algorithm that checks deterministically in polynomial time for a given propositional formula F, whether F lies in DNF-SAT.

(b) Let $F = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a propositional formula in 3-CNF, such that in each clause each variable occurs at most once. Prove the following statement: There is a truth assignment, such that at least 7/8 of the clauses of F evaluate to true.

Hint: Let n be the number of variables of F. With $\mathcal{B}$ we denote a truth assignment of the variables, that is, $\mathcal{B}$ assigns the truth values 0 or 1 to the variables of F. For such a mapping $\mathcal{B}$ we define the functions

$$\chi_i(F, \mathcal{B}) = \begin{cases} 0 & \text{if } C_i \text{ evaluates to 0 under } \mathcal{B} \\ 1 & \text{if } C_i \text{ evalutates to 1 under } \mathcal{B} \end{cases}$$

and the function

$$\chi(F, \mathcal{B}) = \sum_{i=1}^{m} \chi_i(F, \mathcal{B}).$$

Furthermore, let $\mathcal{B}_1, \ldots, \mathcal{B}_{2^n}$ be an enumeration of all possible truth assignments for the n variables. Compute the expectation

$$\mu = \frac{1}{2^n} \sum_{i=1}^{2^n} \sum_{j=1}^{m} \chi_j(F, \mathcal{B}_i).$$

Task 2. Let HC denote the Hamilton circuit problem for undirected graphs and let DHC denote the Hamilton circuit problem for directed graphs. Show that there is a polynomial time reduction from HC to DHC and vice versa.

Task 3. (Recapitulation) Which classes are subsets, respectively proper subsets, of which other classes?

(a) $\mathbf{L}, \mathbf{coNL}, \mathbf{NL}, \mathrm{DSPACE}(\log^4(n)), \mathbf{co}\mathrm{DSPACE}(\log(n^5))$

(b) $\mathbf{NL}, \mathbf{P}, \mathrm{DTIME}(n^2), \mathbf{NEXP}, \mathbf{EXP}, \mathrm{NTIME}(n!)$

(c) $\mathbf{NP}, \mathrm{NSPACE}(3n^5 + n^4 + 1), \mathbf{NPSPACE}, \mathbf{PSPACE}$