Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey

Exercise 3

Task 1
Sort the array $[19, 9, 1, 6, 4, 17, 3, 18]$ using Quicksort (with median-out-of-three).

Task 2
The running time is optimal for Quicksort if the pivot element is the median of the array entries $\{A[1], \ldots, A[n]\}$ (slide 50). On the other hand, the worst-case running time arises when after each call of partition$(A[l...r], p)$, one of the subarrays $A[l...m-1]$ or $A[m+1...r]$ is empty (slide 56). Now, suppose we run a Quicksort algorithm where the partition is unbalanced but maintains a constant 1:10 ratio between the lengths of the subarrays.

- Give a recursion for the running time of this Quicksort algorithm.
- Draw a tree that describes how the elements are distributed in every iteration.
- What is the running time of this Quicksort algorithm?
- (Optional) Have a look at the Akra–Bazzi theorem.

Task 3 (Slides 57 and 62)
Show that the n-th harmonic number H_n satisfies the following inequalities.

$$\ln(n+1) \leq H_n \leq \ln(n) + 1$$

Hint: $\ln(n) = \int_1^n \frac{1}{x}\,\mathrm{d}x$.

Task 4
Show the following two identities for harmonic numbers by induction.

(a) $\sum_{k=1}^n H_k = (n+1)H_n - n$

(b) $\sum_{k=1}^n H_k^2 = (n+1)H_n^2 - (2n+1)H_n + 2n$