# Exercise 4

## Task 1

Sort the array [8, 4, 9, 1, 5, 3, 6, 7] using standard Heapsort and using bottom-up Heapsort. How many comparisons did you make in each case?

## Task 2

Consider the coding functions  $c_1$  and  $c'_2$  (slides 78 and 80) given on  $a_1 \ldots a_t \in \{0, 1\}^*$  by

 $c_1(a_1 \dots a_t) = a_1 0 a_2 0 \dots a_{t-1} 0 a_t 1$  and  $c'_2(a_1 \dots a_t) = c_1(bin(\lceil \log_2(n) \rceil - t)) a_1 \dots a_t.$ 

For an input list of length n = 6 we get the following code of the sink paths from Heapsort:

#### 1001010011111010011100101001110010.

All sink paths are encoded using  $c'_2$ . What is the input list?

#### Task 3

Prove Jensen's inequality (slide 9).

# Task 4

Does there exist a comparison-based sorting algorithm with the following property?

There is a constant c > 0 such that for all n the proportion of all input lists of length n on which the algorithm makes at most  $c \cdot n$  comparisons is at least  $\frac{1}{2^n}$ .