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Lehrstuhl Theoretische Informatik
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Exercise 6

Task 1
Which of the following pairs (E, U) is a subset system or a matroid, respectively?

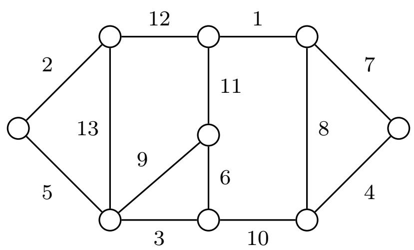
1. $E = \{1, 2, 3\}$ and $U = \big\{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2, 3\}\big\}$.
2. $E = \{1, 2, 3\}$ and $U = \big\{\emptyset, \{1\}, \{2\}, \{3\}, \{2, 3\}\big\}$.
3. E is a finite set and $U = \{A \subseteq E \mid |A| \leq k\}$ for some $k \in \mathbb{N}$.
4. E is a finite subset of $\mathbb{R}^2$ and U consists of all linearly independent subsets of E.

Task 2
Let $T = (V, E)$ be a finite tree (slide 121) with $|V| \geq 1$. Prove the following statements.

1. There exists a vertex $v \in V$ with at most one neighbour, i.e., $\{v, u_1\}, \{v, u_2\} \in E$ implies $u_1 = u_2$. In other words, every finite non-empty tree T has a leaf.
2. The equality $|V| - |E| = 1$ holds.

Task 3
Show that every finite connected graph $G = (V, E)$ has a spanning subtree (slide 122). Moreover, use the equality from Task 2 to conclude that G has a circuit if $|E| \geq |V| \geq 1$.

Task 4
Compute a spanning subtree of maximal weight for the following edge-weighted graph using Kruskal's algorithm (slide 126).



How does the result change, when you compute a spanning subtree of minimal weight?