

Exercise 1

Task 1

Prove or disprove the following statements:

- (a) $n \log n \in \mathcal{O}(\sqrt{n} \cdot n)$
- (b) $n^{k+1} \in \mathcal{O}(n^k)$, $k \in \mathbb{N}$
- (c) $n^n \in \Omega(2^n)$
- (d) $n - \log(n) \in o(n)$
- (e) $k + \ell \cdot (-1)^n \in \Theta(1)$, $k, \ell \in \mathbb{N}$, $k > \ell$
- (f) $n^3 \in \omega(n^2 \sin(n!))$

Task 2

Prove that $T(n) = a_0 + a_1n + \dots + a_kn^k$ is $O(n^k)$ using the formal definition of the Big-O notation.

Task 3

Let $f: \mathbb{N} \rightarrow \mathbb{N}$ with $f(n) \in \Theta(n)$. Prove or disprove the following statements:

- (a) $f(n)^k \in \Theta(n^k)$ for all $k \in \mathbb{N}$, $k \geq 1$
- (b) $2^{f(n)} \in \Theta(2^n)$

Task 4

Use the Master Theorem to determine the asymptotic growth of the following functions:

- (a) $T_1(n) = 99 \cdot T_1\left(\frac{n}{100}\right) + 12n$
- (b) $T_2(n) = T_2\left(\frac{2n}{3}\right) + 1$
- (c) $T_3(n) = 16 \cdot T_3\left(\frac{n}{2}\right) + n^4$
- (d) $T_4(n) = 9 \cdot T_4\left(\frac{n}{3}\right) + n$

Task 5

Give a recursive equation for the running time of the algorithm for f . Use the Master Theorem I to compute the running time of the algorithm.

```
function  $f(n$  : integer)
  let  $m$  be the smallest number of the form  $5^k$  with  $5^k \geq n$ ;
  if  $m = 1$  then print(goodbye)
  else
    for  $i = 1$  to  $m^2$  do
      for  $j = 1$  to  $m^2$  do
        print(hello)
      endfor
    endfor
     $m := m/5$ ;
    for  $i = 1$  to 9 do
       $f(m)$ 
    endfor
  endif
endfunction
```