Exam for „Algorithms I“

WS 2024/25 / February 17, 2025

First name: _____

Second name: _____

Matriculation number: _____

task	max. points	points achieved
1	7	
2	5	
3	6	
4	5	
5	7	
6	6	
7	6	
Σ	42	

Important information

- Duration of the exam: **60 minutes**.
- Tools: You are allowed to use a handwritten sheet of paper (size DIN A4). Both sides of the sheet of paper can be handwritten.
- Write with an indelible pen. Do not write in red paint.
- Check the exam you have been given for completeness: **7 tasks** on 9 pages.
- Enter your name and matriculation number in the appropriate fields on each sheet.
- Write your solutions in the spaces provided. If there is not enough space in a field, use the back of the corresponding sheet and indicate this on the front. If there is still not enough space, you can ask the supervisor for additional sheets of paper.
- Please write clearly. Illegible answers are invalid.
- Any attempt to cheat will result in immediate exclusion and failure. There will be no advance warning.
- All electronic devices must be switched off before the exam or at the latest now.

Task 1. (7 Points)

- Which of the following statements hold? You do not have to prove your answers.

  (1) $n^n \in \Omega(n!)$ (2) $3^{2n} \in \mathcal{O}(2^{3n})$ (3) $\frac{1}{n} \log n \in o(1)$

- Using the Master Theorems, determine the asymptotic growth (in Θ-notation) of the functions f and g subject to the following recursions.

  (4) $f(n) = 4\, f(n/2) + n^2$ (5) $g(n) = 9\, g(n/3) + n\sqrt{n}$

Task 2. (5 Points)

Recall that the height of a tree is the number of edges along a longest path from the root of the tree to a leaf. Prove that, for every $h \in \mathbb{N}$, there are at least 2^h distinct binary trees of heigth h.

Task 3. (6 Points)

- Write down the pseudocode for the procedure **counting-sort**.

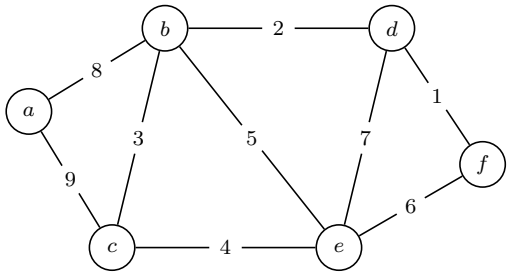
- Sort the following array of numbers using Radix Sort (in base 10).

$$[568, 416, 538, 857, 976, 462, 389, 543]$$

Write down the new array after each call to **counting-sort**.

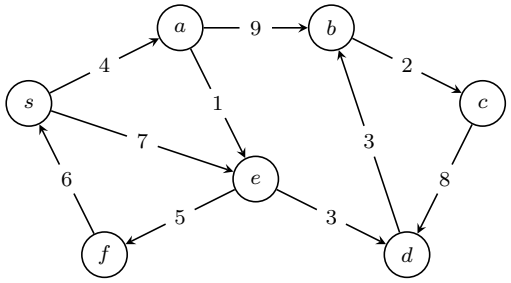
Task 4. (5 Points)

Compute a spanning subtree of *minimal* weight using Kruskal's algorithm for the following graph. Show the edge selected in each step or indicate if no edge is selected.



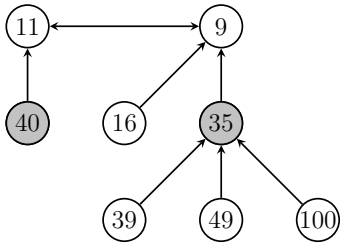
Task 5. (7 Points)

Use Dijkstra's algorithm to compute all shortest paths starting at node s in the graph below. Show the values of the program variables B, R, U, p, D after each iteration of the main **while**-loop of Dijkstra's algorithm.



Task 6. (6 Points)

The following Fibonacci heap is given.



Perform the following sequence of operations on the above Fibonacci heap.

(1) **decrease-key**(node with key 100, 12)

(2) **delete-min**

(3) **delete-min**

Task 7. (6 Points)

Let us consider directed graphs $G = (V, E)$ on the vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and the property that $i < j$ for every edge from v_i to v_j in G. On input of such a graph G, we want to compute the number of distinct paths from v_1 to v_n.

- Give a brief description of a dynamic programming solution for this problem.

- Compute the number of distinct paths from v_1 to v_7 in the following graph.