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Exercise 3

Task 1 (Savitch's Theorem)**.** Recall that Savitch's Theorem states that for all functions $s(n) \in \Omega(\log n)$ we have $\mathbf{NSPACE}(s(n)) \subseteq \mathbf{DSPACE}(s^2(n))$.

Compute the running time of the algorithm (slide 35) used in the proof of this theorem, i.e., compute the time required in order to deterministically simulate a nondeterministic Turing machine with quadratic space increase.

Task 2. Compare the following pairs of complexity classes. Which of these classes is a subset of the other class? Give reasons for your answer.

- $\mathbf{NTIME}(n^2)$ and $\mathbf{DSPACE}(n^3)$
- $\mathbf{DTIME}(n^2+1)$ and $\mathbf{DTIME}(3n^2+\log^4 n)$

Task 3. Show that the function $f_1(n) = n^2$ is time and space constructible by describing appropriate Turing machines. Furthermore, construct Turing machines that show that $f_2(n) = 2^n$ and $f_3(n) = n!$ are space constructible.

Task 4. Let $f, g\colon \mathbb{N} \to \mathbb{N}$ be two time (resp. space) constructible functions. Which of the following statements is true? Justify your answers.

- The sum $f + g$ is also time (resp. space) constructible.
- The product $f \cdot g$ is also time (resp. space) constructible.
- The composition $g \circ f$ is also time (resp. space) constructible.

Moreover, show that every polynomial $p(x) \in \mathbb{N}[x]$ with non-negative integer coefficients is both time constructible and space constructible.