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Exercise 4

Task 1. A *cycle* in a directed graph is a path $v_0, \ldots, v_n$, such that $v_0 = v_n$ and $v_i \neq v_j$ for all other nodes v_i and v_j. A directed graph is called *acyclic*, if it does not contain a cycle. Consider the problem ACYCLIC defined as follows.

Input: A directed graph G.
Question: Is G acyclic?

Does the problem ACYCLIC belong to **NL**?

Task 2. Show that the theorem of Immerman and Szelepcsényi (slide 51) is equivalent to **NL** = **coNL** by giving a suitable padding argument.

Task 3. Consider the complexity classes

$$\mathbf{EXP} = \bigcup_{k \in \mathbb{N}} \mathbf{DTIME}(2^{n^k}) \quad \text{and} \quad \mathbf{NEXP} = \bigcup_{k \in \mathbb{N}} \mathbf{NTIME}(2^{n^k}).$$

Prove that **P** = **NP** implies **EXP** = **NEXP** using the padding technique.

Task 4. Let $L \subsetneq \Sigma^*$ be any language and let $f(n) = n^k$. Show that

$$\mathsf{Pad}_f(L) \leq_m^{\log} L.$$