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Exercise 6

Task 1. Let DNF-SAT be the set of satisfiable propositional formulas in disjunctive normal form. Find an algorithm checking deterministically in polynomial time whether a given propositional formula F belongs to DNF-SAT.

Task 2. Let $F = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a propositional formula in 3-CNF such that in each clause each variable occurs at most once. Prove that there exists a truth assignment under which at least $\frac{7}{8}$ of the clauses of F evaluate to true.

Hint: Suppose that the truth assignment $\mathcal{B}$ is chosen uniformly at random. Compute the expected value of $\chi(\mathcal{B}) = \chi_1(\mathcal{B}) + \chi_2(\mathcal{B}) + \ldots + \chi_m(\mathcal{B})$ where

$$\chi_i(\mathcal{B}) = \begin{cases} 0 & \text{if } C_i \text{ evaluates to false under } \mathcal{B} \\ 1 & \text{if } C_i \text{ evaluates to true under } \mathcal{B}. \end{cases}$$

Task 3. Let NAE-3-SAT be the set of propositional formulas in 3-CNF such that there exists a truth assignment under which in each clause the values of the literals are *not all equal*. For example, $(x \vee x \vee \neg x) \in$ NAE-3-SAT but $(x \vee x \vee \neg x) \wedge (y \vee y \vee y) \notin$ NAE-3-SAT. Show that NAE-3-SAT is **NP**-complete.

Hint: To show **NP**-hardness, first reduce 3-SAT to NAE-4-SAT (which is defined analogously) by introducing a common literal to all clauses and then reduce NAE-4-SAT to NAE-3-SAT by splitting clauses (slide 109).

Task 4. (Recapitulation) Compare the following complexity classes.

(a) $\mathbf{L}, \mathbf{NL}, \mathbf{DSPACE}(\log^4(n)), \mathbf{coNL}, \mathbf{coDSPACE}(\log(n^5))$

(b) $\mathbf{NL}, \mathbf{NTIME}(n!), \mathbf{P}, \mathbf{DTIME}(n^3), \mathbf{NEXP}, \mathbf{EXP}$

(c) $\mathbf{NPSPACE}, \mathbf{NP}, \mathbf{NSPACE}(3n^5 + n^4 + 1), \mathbf{PSPACE}$