# Exercise 2

## Task 1

Sort the array [5, 2, 4, 16, 1, 3, 2, 6] using Mergesort.

#### Task 2

Calculate 3255 · 6789 using the algorithm of Karatsuba. You do not have to use base 2.

#### Task 3

Use the algorithm of Strassen to calculate the following matrix product:

$$\begin{pmatrix} 5 & -5 \\ 5 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

### Task 4

Professor Caesar wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's algorithm. His algorithm will use the divide-and-conquer method, dividing each matrix into pieces of size  $n/16 \times n/16$ , and the divide and combine steps together will take  $\Theta(n^2)$  time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen's algorithm. If his algorithm creates a subproblems then the recurrence for the running time T(n) becomes  $aT(n/16) + \Theta(n^2)$ . What is the largest integer value for a for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm?

### Task 5

Show that a binary tree with N leaves has at least height  $\log_2(N)$ .