Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey

Algorithmics 1
WS 2025/26

Exercise 5

Task 1
Let $(x_1, y_1), \ldots, (x_n, y_n)$ be n points in the plane $\mathbb{R}^2$. Find a line g parallel to the x-axis in time $\mathcal{O}(n)$, such that the sum of the distances between g and the points is minimal. Prove that your line is indeed optimal.

Task 2
Which of the following pairs is a subset system, respectively matroid?

(a) $(\{1,2,3\}, \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2,3\}\})$

(b) $(\{1,2,3\}, \{\emptyset, \{1\}, \{2\}, \{3\}, \{2,3\}\})$

(c) (E, U), where E is a finite set and $U = \{A \subseteq E \mid |A| \leq k\}$ for a $k \in \mathbb{N}$.

(d) (E, U), where E is a finite subset of a vector space (for instance $\mathbb{R}^2$) and U consists of all linearly independent subsets of E.

(e) (E, U), where E is the set of edges from an undirected graph G and U the subsets of edges that do not form cycles in the graph.

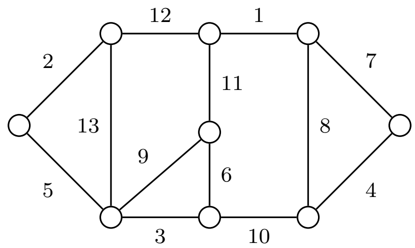
Task 3

(a) Show that for each tree $T = (V, E)$ with $|V| > 0$ we have $|E| = |V| - 1$.

(b) Show that every finite connected graph has a spanning subtree.

Task 4
Show that there is no graph with exactly 2 spanning trees.
Hint: Consider any graph with non-empty set of vertices. Prove that there are only three types of graphs: with 0, with 1 or with at least 3 different spanning trees.

Task 5
Compute a spanning subtree of maximal weight using Kruskal's algorithm for the following graph:



How does the result change, when you want to compute a spanning subtree of minimal weight?