

Exercise 1

Task 1

Prove or disprove the following statements:

- (a) $n \log n \in \mathcal{O}(n^2)$
- (b) $n^2 \in \mathcal{O}(n)$
- (c) $n^n \in \Omega(2^n)$
- (d) $n - \log n \in o(n)$
- (e) $2 + (-1)^n \in \Theta(1)$
- (f) $n! \in \omega(2^n)$

Task 2

Prove that $T(n) = a_0 + a_1n + \dots + a_kn^k$ is $O(n^k)$ using the formal definition of the Big-O notation.

Task 3

Let $f: \mathbb{N} \rightarrow \mathbb{N}$ with $f(n) \in \Theta(n)$. Prove or disprove the following statements:

- (a) $f(n)^k \in \Theta(n^k)$ for all $k \in \mathbb{N}$, $k \geq 1$
- (b) $2^{f(n)} \in \Theta(2^n)$

Task 4

Use the Master Theorem to determine the asymptotic growth of the following functions:

- (a) $T_1(n) = 99 \cdot T_1\left(\frac{n}{100}\right) + 12n$
- (b) $T_2(n) = T_2\left(\frac{2n}{3}\right) + 1$
- (c) $T_3(n) = 16 \cdot T_3\left(\frac{n}{2}\right) + n^4$
- (d) $T_4(n) = 9 \cdot T_4\left(\frac{n}{3}\right) + n$