

Exercise 2

Task 1. Show that the following languages are in **L** by constructing Turing machines for these languages.

1. $L_1 = \{a^n b^n c^n \mid n \geq 1\}$
2. $L_2 = \{v\$v \mid v \in \Sigma^*\}$

Task 2. Prove the statement on slide 22.

Task 3 (Nondeterministic Logspace). A directed graph $G = (V, E)$ is called *strongly connected*, if for all pairwise distinct nodes v_i, v_j there is a directed path from v_i to v_j .

Does the problem

Input: A directed graph $G = (V, E)$.

Question: Is G strongly connected?

belong to the complexity class **NL**?

Task 4 (Deterministic Logspace). Does the problem

Input: An integer $N \in \mathbb{N}$ in *unary* encoding.

Question: Is N a prime number?

belong to the complexity class **L**?

Task 5 (Savitch's Theorem). Compute the running time of the algorithm on slide 35. Hence, compute the time required in order to simulate a non-deterministic Turing machine in a deterministic way with quadratic space increase.

Reminder: Savitch's Theorem states that for functions $s(n) \in \Omega(\log n)$ we have $\text{NSPACE}(s(n)) \subseteq \text{DSPACE}(s^2(n))$.