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Exercise 4

Task 1. Show that the theorem of Immerman and Szelepcsényi (slide 51) is equivalent to $\mathbf{NL} = \mathbf{coNL}$ by giving a suitable padding argument.

Task 2. Consider the complexity classes

$$\mathbf{EXP} = \bigcup_{k \in \mathbb{N}} \mathbf{DTIME}(2^{n^k}) \quad \text{and} \quad \mathbf{NEXP} = \bigcup_{k \in \mathbb{N}} \mathbf{NTIME}(2^{n^k}).$$

Prove that $\mathbf{P} = \mathbf{NP}$ implies $\mathbf{EXP} = \mathbf{NEXP}$ using the padding technique.

Task 3. Let $L \subsetneq \Sigma^*$ be any language and let $f(n) = n^k$. Show that

$$\mathsf{Pad}_f(L) \leq_m^{\log} L.$$

Task 4. Which of the following statements are true provided that $L_1 \leq_m^{\log} L_2$?

- If L_1 is in $\mathbf{P}$, then L_2 is in $\mathbf{P}$.
- If L_1 is $\mathbf{NL}$-hard, then L_2 is $\mathbf{NL}$-hard.
- If L_2 is in $\mathbf{L}$, then L_1 is in $\mathbf{L}$.
- If L_2 is $\mathbf{NP}$-complete, then L_1 is $\mathbf{NP}$-complete.